Section 6.1 Exercises

In Exercises 1–6, find an antiderivative for the function. Confirm your answer by differentiation.

1. $x^2 - 2x + 1$
2. $-3x^{-4}$
3. $x^2 - 4\sqrt{x}$
4. $8 + \csc x \cot x$
5. e^{4x}
6. $\dfrac{1}{x+3}$

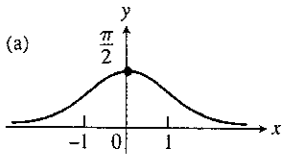
In Exercises 7–24, evaluate the integral.

7. $\displaystyle\int (x^5 - 6x + 3)\,dx$
8. $\displaystyle\int (-x^{-3} + x - 1)\,dx$
9. $\displaystyle\int \left(e^{t/2} - \frac{5}{t^2}\right) dt$
10. $\displaystyle\int \frac{4}{3}\sqrt[3]{t}\,dt$
11. $\displaystyle\int \left(x^3 - \frac{1}{x^3}\right) dx$
12. $\displaystyle\int \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\right) dx$
13. $\displaystyle\int \frac{1}{3}x^{-2/3}\,dx$
14. $\displaystyle\int (3\sin x - \sin 3x)\,dx$
15. $\displaystyle\int \frac{\pi}{2}\cos\frac{\pi x}{2}\,dx$
16. $\displaystyle\int 2\sec t \tan t\,dt$
17. $\displaystyle\int \left(\frac{2}{x-1} + \frac{1}{x}\right) dx$
18. $\displaystyle\int \left(\frac{1}{x-2} + \sin 5x - e^{-2x}\right) dx$
19. $\displaystyle\int 5\sec^2 5r\,dr$
20. $\displaystyle\int \csc^2 7t\,dt$
21. $\displaystyle\int \cos^2 x\,dx$ (*Hint:* $\cos^2 x = \dfrac{1 + \cos 2x}{2}$)
22. $\displaystyle\int \sin^2 x\,dx$ (*Hint:* $\sin^2 x = \dfrac{1 - \cos 2x}{2}$)
23. $\displaystyle\int \tan^2\theta\,d\theta$
24. $\displaystyle\int \cot^2 t\,dt$

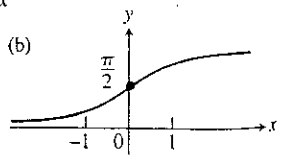
In Exercises 25 and 26, **(a)** determine which graph shows the solution of the initial value problem without actually solving the problem. **(b) Writing to Learn** Explain why you eliminated two of the possibilities.

25. $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}, \quad y(0) = \dfrac{\pi}{2}$

(a)



(b)



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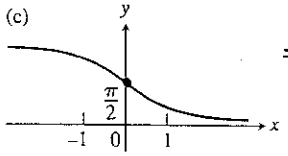
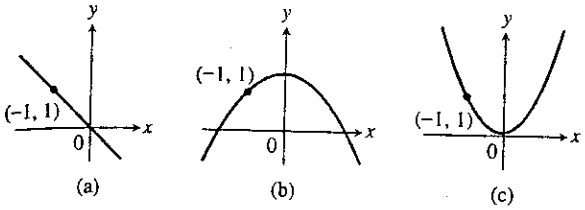


26. $\frac{dy}{dx} = -x, \quad y(-1) = 1$



In Exercises 27–30, solve the initial value problem. Support your answer by overlaying your solution on a slope field for the differential equation.

27. $\frac{dy}{dx} = 2x - 1, \quad y(2) = 0$

28. $\frac{dy}{dx} = \frac{1}{x^2} + x, \quad y(2) = 1$

29. $\frac{dy}{dx} = \sec^2 x, \quad y(\pi/4) = -1$

30. $\frac{dy}{dx} = x^{-2/3}, \quad y(-1) = -5$

In Exercises 31–38, solve the initial value problem.

31. $\frac{dy}{dx} = 9x^2 - 4x + 5, \quad y(-1) = 0$

32. $\frac{dy}{dx} = \cos x + \sin x, \quad y(\pi) = 1$

33. $\frac{dy}{dt} = 2e^{-t}, \quad y(\ln 2) = 0$

34. $\frac{dy}{dx} = \frac{1}{x}, \quad y(e^3) = 0$

35. $\frac{d^2y}{d\theta^2} = \sin\theta, \quad y(0) = -3, \quad y'(0) = 0$

36. $\frac{d^2y}{dx^2} = 2 - 6x, \quad y(0) = 1, \quad y'(0) = 4$

37. $\frac{d^3y}{dt^3} = \frac{1}{t^3}, \quad y(1) = 1, \quad y'(1) = 3, \quad y''(1) = 2$

38. $\frac{d^4y}{d\theta^4} = \sin\theta + \cos\theta, \quad y(0) = -3, \quad y'(0) = -1,$
$y''(0) = -1, \quad y^{(3)}(0) = -3$

In Exercises 39–42, the velocity $v = ds/dt$ or acceleration $a = dv/dt$ of a body moving along a coordinate line is given. Find the body's position s at time t.

39. $v = 9.8t + 5, \quad s(0) = 10$

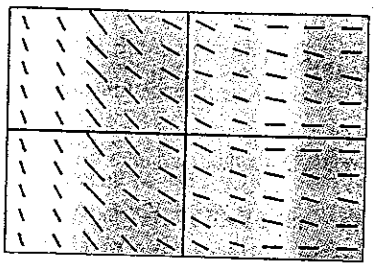
40. $v = \sin \pi t, \quad s(1) = 0$

41. $a = 32, \quad s(0) = 0, \quad v(0) = 20$

42. $a = \cos t, \quad s(0) = 1, \quad v(0) = -1$

In Exercises 43 and 44, *work in groups of two or three.* Draw a possible graph for the function f with the given slope field that satisfies the stated condition.

43. $f(0) = 0$



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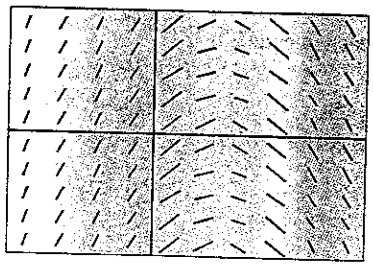
44. $f(-1) = -2$



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In Exercises 45–48, confirm the integration formula by differentiation.

45. $\int \frac{dx}{1 + x^2} = \tan^{-1} x + C$

46. $\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + C$

47. $\int \frac{dx}{|x|\sqrt{x^2 - 1}} = \sec^{-1} x + C$

48. $\int \frac{dx}{\sqrt{1 - x^2}} = -\cos^{-1} x + C$

Exploration

49. Let $\frac{dy}{dx} = x - \frac{1}{x^2}$.

(a) Find a solution to the differential equation in the interval $(0, \infty)$ that satisfies $y(1) = 2$.

(b) Find a solution to the differential equation in the interval $(-\infty, 0)$ that satisfies $y(-1) = 1$.

(c) Show that the following piecewise function is a solution to the differential equation for any values of C_1 and C_2.

$$y = \begin{cases} \frac{1}{x} + \frac{x^2}{2} + C_1, & x < 0 \\ \frac{1}{x} + \frac{x^2}{2} + C_2, & x > 0 \end{cases}$$

(d) Choose values for C_1 and C_2 so that the solution in part (c) agrees with the solutions in parts (a) and (b).

(e) Choose values for C_1 and C_2 so that the solution in part (c) satisfies $y(2) = -1$ and $y(-2) = 2$. ■