Exercises

Random Selection **In Exercises 1–8, determine the number of ways a computer can randomly generate one or more such integers from 1 through 12.**

1. An odd integer
2. An even integer
3. A prime integer
4. An integer that is greater than 7
5. An integer that is divisible by 4
6. An integer that is divisible by 3
7. Two integers whose sum is 10
8. Two *distinct* integers whose sum is 10

9. ***Entertainment Systems*** A customer can choose one of four amplifiers, one of six compact disc players, and one of eight speaker models for an entertainment system. Determine the number of possible system configurations.

10. ***Computer Systems*** A customer in a computer store can choose one of three monitors, one of two keyboards, and one of seven computers. If all the choices are compatible, determine the number of possible system configurations.

11. ***Job Applicants*** A college needs two additional faculty members: a chemist and a statistician. In how many ways can these positions be filled if there are three applicants for the chemistry position and six applicants for the statistics position?

12. ***Course Schedule*** A college student is preparing a course schedule for the next semester. The student must select one of two mathematics courses, one of three science courses, and one of five courses from the social sciences and humanities. How many schedules are possible?

13. ***Periodic Table*** You are taking a chemistry test and are asked to list the first ten elements *in order* as they appear in the periodic table of elements. Suppose you have no idea of the correct order and simply guess. In how many different orders could you list the elements?

14. ***True-False Exam*** In how many ways can a ten-question true-false exam be answered? (Assume that no questions are omitted.)

15. ***Toboggan Ride*** Four people are lining up for a ride on a toboggan, but only two of the four are willing to take the first position. With that constraint, in how many ways can the four people be seated on the toboggan?

16. ***Taking a Trip*** Four people are taking a long trip in a four-seat car. Three of the people agree to share the driving. In how many different arrangements can the four people sit?

17. ***License Plate Numbers*** In a certain state the automobile license plates consist of two letters followed by a four-digit number. How many distinct license plate numbers can be formed?

18. ***License Plate Numbers*** In a certain state the automobile license plates consist of two letters followed by a four-digit number. To avoid confusion between "O" and "zero" and "I" and "one," the letters "O" and "I" are not used. How many distinct license plate numbers can be formed?

19. ***Three-Digit Numbers*** How many three-digit numbers can be formed under the following conditions?
    (a) The leading digit cannot be zero.
    (b) The leading digit cannot be zero and no repetition of digits is allowed.
    (c) The leading digit cannot be zero and the number must be a multiple of 5.
    (d) The number is at least 400.

20. ***Four-Digit Numbers*** How many four-digit numbers can be formed under the following conditions?
    (a) The leading digit cannot be zero.
    (b) The leading digit cannot be zero and no repetition of digits is allowed.
    (c) The leading digit cannot be zero and the number must be less than 5000.
    (d) The leading digit cannot be zero and the number must be even.

21. ***Combination Lock*** A combination lock will open when the right choice of three numbers (from 1 to 40, inclusive) is selected. How many different lock combinations are possible?

22. ***Telephone Numbers*** In 1997, the Commonwealth of Massachusetts had three area codes: one for the Boston metropolitan area, one for the rest of eastern Massachusetts, and one for western Massachusetts. Using the information about telephone numbers in Example 4, how many telephone numbers could the phone system have accommodated in the state of Massachusetts?

23. ***Concert Seats*** Three couples have reserved seats in a given row for a concert. In how many different ways can they be seated if
   (a) there are no seating restrictions?
   (b) the two members of each couple wish to sit together?

24. ***Single File*** In how many orders can five girls and three boys walk through a doorway single file if
   (a) there are no restrictions?
   (b) the girls walk through before the boys?

In Exercises 25–30, evaluate $_nP_r$ using the formula.

25. $_4P_4$
26. $_5P_5$
27. $_8P_3$
28. $_{20}P_2$
29. $_5P_4$
30. $_7P_4$

In Exercises 31 and 32, solve for n.

31. $14 \cdot {}_nP_3 = {}_{n+2}P_4$
32. $_nP_5 = 18 \cdot {}_{n-2}P_4$

In Exercises 33–38, evaluate using a graphing utility.

33. $_{20}P_6$
34. $_{100}P_5$
35. $_{120}P_4$
36. $_{10}P_8$
37. $_{20}C_4$
38. $_{10}C_7$

39. ***Posing for a Photograph*** In how many ways can five children line up in a row?

40. ***Riding in a Car*** In how many ways can four people sit in a four-passenger car?

41. ***Morse Code*** In Morse code, all characters are transmitted using a sequence of *dots* and *dashes*.
   (a) How many different characters can be formed with a sequence of four symbols, each of which is a dot or dash?
   (b) How many can be formed with a sequence of one, two, or three symbols?

42. ***Assembly Line Production*** Four processes are involved in assembling a certain product, and they can be performed in any order. The management wants to test each order to determine which is the least time consuming. How many different orders will have to be tested?

In Exercises 43–46, find the number of distinguishable permutations of the group of letters.

43. A, A, G, E, E, E, M
44. B, B, B, T, T, T, T, T
45. A, L, G, E, B, R, A
46. M, I, S, S, I, S, S, I, P, P, I
47. Write all permutations of the letters A, B, C, and D.
48. Write all the permutations of the letters A, B, C, and D if the letters B and C must remain between the letters A and D.
49. Write all the possible selections of two letters that can be formed from the letters A, B, C, D, E, and F. (The order of the two letters is not important.)
50. Write all the possible selections of three letters that can be formed from the letters A, B, C, D, E, and F. (The order of the three letters is not important.)
51. ***Forming an Experimental Group*** In order to conduct a certain experiment, four students are randomly selected from a class of 20. How many different groups of four students are possible?
52. ***Test Questions*** You can answer any 12 questions from a total of 14 questions on an exam. In how many different ways can you select the questions?
53. ***Lottery Choices*** There are 40 numbers in a particular state lottery. In how many ways can a player select six of the numbers?
54. ***Lottery Choices*** There are 50 numbers in a particular state lottery. In how many ways can a player select six of the numbers?
55. ***Number of Subsets*** How many subsets of five elements can be formed from a set of 100 elements?
56. ***Number of Subsets*** How many subsets of six elements can be formed from a set of 80 elements?
57. ***Geometry*** Three points that are not on a line determine three lines. How many lines are determined by nine points, no three of which are on a line?
58. ***Defective Units*** A shipment of 25 television sets contains three defective units. In how many ways can a vending company purchase four of these units and receive (a) all good units, (b) two good units, and (c) at least two good units?

59. *Job Applicants* An employer interviews 12 people for four openings in the company. Five of the 12 people are women. If all 12 are qualified, in how many ways can the employer fill the four positions if (a) the selection is random and (b) exactly two women are selected?

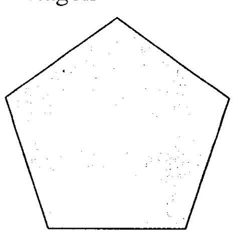
60. *Poker Hand* Five cards are selected from an ordinary deck of 52 playing cards. In how many ways can you get a full house? (A full house consists of three of one kind and two of another. For example, 8-8-8-5-5 and K-K-K-10-10 are full houses.)

61. *Forming a Committee* Four people are to be selected at random from a group of four couples. In how many ways can this be done, given the following conditions?
   (a) There are no restrictions.
   (b) The group must have at least one couple.
   (c) Each couple must be represented in the group.

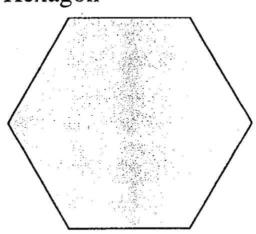
62. *Interpersonal Relationships* The complexity of the interpersonal relationships increases dramatically as the size of a group increases. Determine the number of different two-person relationships in a group of people of size (a) 3, (b) 8, (c) 12, and (d) 20.

In Exercises 63–66, find the number of diagonals of the polygon. (A line segment connecting any two nonadjacent vertices is called a *diagonal* of the polygon.)

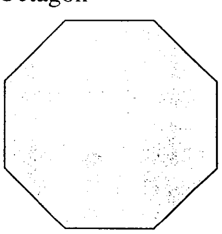
63. Pentagon



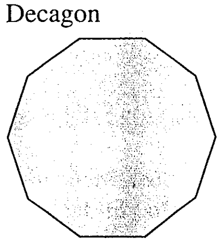
64. Hexagon



65. Octagon



66. Decagon



Synthesis

***True or False?* In Exercises 67–69, determine whether the statement is true or false. Justify your answer.**

67. The number of pairs of letters that can be formed from any of the first 13 letters in the alphabet (A–M) is an example of a permutation.

68. The number of permutations of n elements can be derived by using the Fundamental Counting Principle.

69. ${}_nP_r > {}_nC_r$ always.

70. *Think About It* Can your calculator evaluate ${}_{100}P_{80}$? If not, explain why.

71. *Writing* Explain in words the meaning of ${}_nP_r$.

72. What is the relationship between ${}_nC_r$ and ${}_nC_{n-r}$?

73. Without calculating the numbers, determine which of the following is greater. Explain.
   (a) The combinations of 10 elements taken 6 at a time
   (b) The permutations of 10 elements taken 6 at a time

In Exercises 74–77, prove the identity.

74. ${}_nP_{n-1} = {}_nP_n$

75. ${}_nC_n = {}_nC_0$

76. ${}_nC_{n-1} = {}_nC_1$

77. ${}_nC_r = \dfrac{{}_nP_r}{r!}$

Review

In Exercises 78–81, solve the equation. Round your answer to two decimal places, if necessary.

78. $\sqrt{x-3} = x - 6$

79. $\dfrac{4}{t} + \dfrac{3}{2t} = 1$

80. $\log_2(x - 3) = 5$

81. $e^{x/3} = 16$

In Exercises 82–85, use Cramer's Rule to solve the system of equations.

82. $\begin{cases} -5x + 3y = -14 \\ 7x - 2y = 2 \end{cases}$

83. $\begin{cases} 8x + y = 35 \\ 6x + 2y = 10 \end{cases}$

84. $\begin{cases} -3x - 4y = -1 \\ 9x + 5y = -4 \end{cases}$

85. $\begin{cases} 10x - 11y = -74 \\ -8x - 4y = 8 \end{cases}$

In Exercises 86–89, use the Binomial Theorem to expand and simplify the expression.

86. $(x - 4)^3$

87. $(x - 1)^6$

88. $(x^2 + 4)^5$

89. $(3x - y)^4$

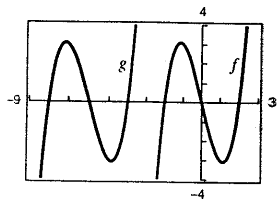
61. $3x^2 + 3xh + h^2,\ h \neq 0$

63. $\dfrac{\sqrt{x+h} - \sqrt{x}}{h} = \dfrac{1}{\sqrt{x+h} + \sqrt{x}},\ h \neq 0$

65. -4 **67.** $2035 + 828i$ **69.** 1 **71.** 1.172

73. 510,568.785

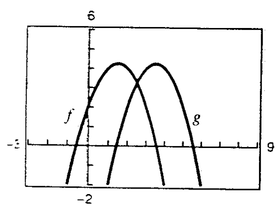
75.



g is shifted 6 units left of f.

$g(x) = x^3 + 18x^2 + 104x + 192$

77.

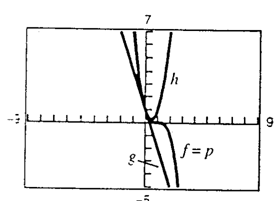


g is shifted 2 units right of f.

$g(x) = -x^2 + 7x - 8$

79. (a) 792 (b) 36 (c) 792 (d) 12

81.

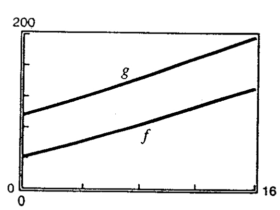


$p(x)$ is the expansion of $f(x)$.

83. 0.273 **85.** 0.171

87. (a) $g(t) = 0.0348t^2 + 5.8043t + 95.588$

(b)



89. False. The correct term is $126{,}720x^4y^8$.

91. The first and last numbers in each row are 1. Every other number in each row is formed by adding the two numbers immediately above the number.

93. $n + 1$ terms **95. and 97.** Answers will vary.

99. $g(x)$ is shifted 8 units up from $f(x)$.

101. $g(x)$ is the reflection of $f(x)$ in the y-axis.

103. $\begin{bmatrix} -2 & -1 & -3 \\ 8 & 0 & 4 \\ -1 & -2 & 7 \end{bmatrix}$ **105.** $\begin{bmatrix} 6 & 11 & 15 \\ -30 & -2 & -16 \\ 13 & 10 & -33 \end{bmatrix}$

107. $\begin{bmatrix} 9 & 11 & 12 \\ -13 & -25 & -5 \\ -5 & -18 & -6 \end{bmatrix}$ **109.** $\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

Section 9.6 *(page 671)*

1. 6 **3.** 5 **5.** 3 **7.** 9 **9.** 192 **11.** 18

13. 3,628,800 **15.** 12 **17.** 6,760,000

19. (a) 900 (b) 648 (c) 180 (d) 600

21. 64,000 **23.** (a) 720 (b) 48 **25.** 24

27. 336 **29.** 120 **31.** $n = 5$ or $n = 6$

33. 27,907,200 **35.** 197,149,680 **37.** 4845

39. 120 **41.** (a) 16 (b) 14 **43.** 420 **45.** 2520

47. ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, CABD, CADB, DABC, DACB, BCAD, BDAC, CBAD, CDAB, DBAC, DCAB, BCDA, BDCA, CBDA, CDBA, DBCA, DCBA

49. AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF

51. 4845 **53.** 3,838,380 **55.** 75,287,520 **57.** 36

59. (a) 495 (b) 210 **61.** (a) 70 (b) 54 (c) 16

63. 5 **65.** 20

67. False. This is an example of a combination.

69. False. ${}_nP_r = {}_nC_r$ if $r = 1$ or 0.

71. ${}_nP_r$ represents the number of ways to choose and order r elements out of a collection of n elements.

73. (b). Numerous permutations can be made from each combination.

75. and 77. Answers will vary. **79.** $\frac{11}{2}$ **81.** 8.32

83. $(6, -13)$ **85.** $(-3, 4)$

87. $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

89. $81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$

Section 9.7 *(page 682)*

1. $\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6),$
$(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

3. {ABC, ACB, BAC, BCA, CAB, CBA}

5. {(A, B), (A, C), (A, D), (A, E), (B, C), (B, D), (B, E), (C, D), (C, E), (D, E)}

7. $\frac{3}{8}$ **9.** $\frac{7}{8}$ **11.** $\frac{3}{13}$ **13.** $\frac{3}{26}$ **15.** $\frac{1}{9}$ **17.** $\frac{35}{36}$

19. $\frac{1}{6}$ **21.** $\frac{1}{5}$ **23.** $\frac{2}{5}$ **25.** 0.3 **27.** $\frac{2}{3}$